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THE FAST† PROCEDURE FOR PREDICTING TRANSIENT SUBCOOLED TWO-PHASE FLOWS

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Abstract—This paper describes a method of predicting transient, two-phase flows in channels, and presents predictions for several problems. The model is based on a Lagrangian drift-flux formulation of the equations of mass and energy in which the liquid phase can be subcooled. The advantage of the present model over previous models lies in the solution technique, which yields accurate solutions very inexpensively and without problems related to stability. In the technique used, analytical solutions to the differential equations that are valid over limited time and space intervals are used to construct the global solution. The example problems include subcooled boiling, flow reversals and blowdown transients.

INTRODUCTION

The equations describing the instantaneous motion of the fluids and interfaces in multi-phase flow are known (Ishii 1973) but solutions to these equations are not within reach. Much more tractable equations can be obtained by performing certain averages over time and space, by introducing idealizations, and by supplying information (e.g. through correlations) that has been lost in the averaging process. The starting point of most recent analyses of two-phase flow has been the equations of mass, momentum and energy, with time and one-space coordinate as the independent variable. In general, the phase velocities and temperatures are not equal (UVUT) but idealizations may include equal velocity (EV) and/or equal temperature (ET).

One thrust of recent research has been the development of "higher level" codes to solve the equations that embody fewer idealizations and approximations (e.g. UV, UT, 2 or 3 space dimensions). However, in applications where many solutions are required, as in design or when the code is to be used as part of a larger systems code to predict the performance of certain components, economics often dictates the use of a low level code which is inexpensive to run. In these cases, the idealizations and approximations embedded in a code of restricted generality could be checked by a few comparisons with predictions of the more general code, and then the less general code used. The objective of the present study was the development of a solution method and computer code, falling into the latter category, which is extremely attractive in terms of accuracy, economy and stability, and yet sufficiently general to handle many of the problems of practical interest.

The present model is based on the cross-sectionally averaged equations of mass and energy. The two momentum equations are replaced by a prescription of the vapour drift velocity, and by an assumption that the pressure drop along the channel is not significant. The latter approximation makes the model unsuitable for the prediction of pressure-wave propagation. In addition, boundary conditions specified as a pressure difference between the ends of the channel are awkward to handle (Khater *et al.* 1980).

On the positive side, the FAST method treats temporally and spatially variable wall heat flux, as well as pressure and flow transients. The effects of departure from thermal equilibrium and radial conduction in fuel rods and channel walls are accounted for in the model. A really unique feature is the high solution accuracy that can be achieved at low cost and without solution difficulties (e.g. instability). The FAST code and the programmer's manual are presently publicly available (Khater *et al.* 1980).

The FAST method is described in the next section. This is followed by the presentation of the results of several calculations.

†Finite-interval Analytic Solution Technique.

An overview of the method

FAST is an acronym for Finite-interval Analytic Solution Technique. In this method, analytical solutions to the equations of interest are obtained which are valid within discrete intervals of time and space. The total space and time domain over which a solution is desired is divided into discrete intervals of the appropriate size, and the analytic solutions that are valid within each of these subdomains are used to construct the solution over the whole domain.

Variations of this solution method have been independently discovered, applied and reported by several authors. The present work represents an extension of the "Hybrid" method of Inayatullah *et al.* (1977), but the name has been changed to FAST to avoid confusion with other methods. Nijsing & Eifler (1975) described a somewhat similar method for problems in heat conduction, while Spalding (1972) and Raithby & Torrance (1974) developed finite-difference schemes based on locally-valid analytical solutions.

To use such a method, one must be able to find the necessary locally-valid analytical solutions from which the global solution is constructed. The ease with which the desired solutions can be obtained depends greatly on which of several alternative formulations of the conservation equations is used. Also, the degree to which simplifications, that are necessary to obtain the desired solutions, limit the domain of validity of the solution depends on the formulation. With these points in mind, the advantage of starting with equations in Lagrangian coordinates, in which time is the only independent variable, will be appreciated. In particular, the drift-flux formulation of Zuber & Staub (1966), which has already led to several analytical solutions (e.g. Staub & Zuber 1967; Lahey *et al.* 1972; Inayatullah & Nicoll 1976).

The Conservation equations

The mass conservation equations for vapour and liquid respectively are (Khater *et al.* 1980):

$$\frac{\partial}{\partial t}\langle \alpha \rho_v \rangle + \frac{\partial}{\partial z}\langle \alpha \rho_v V_v \rangle = \langle \Gamma \rangle \qquad [1]$$

$$\frac{\partial}{\partial t}\langle (1-\alpha)\rho_L \rangle + \frac{\partial}{\partial z}\langle (1-\alpha)\rho_L V_L \rangle = -\langle \Gamma \rangle \qquad [2]$$

where $\langle\ \rangle$ represents a cross-sectional average, and where α is the void fraction, ρ is the density, V is the velocity and Γ is the rate of vapor generation, t is the time and z is the axial position. The subscript v indicates vapour and L indicates liquid. Ignoring conversion of potential and kinetic energy to internal energy, the energy conservation equations are (Khater *et al.* 1980):

$$\frac{\partial}{\partial t}\langle \alpha \rho_v u_v \rangle + \frac{\partial}{\partial z}\langle \alpha \rho_v V_v i_v \rangle = \langle \Gamma_{uv} \rangle \qquad [3]$$

$$\frac{\partial}{\partial t}\langle (1-\alpha)\rho_L u_L \rangle + \frac{\partial}{\partial z}\langle (1-\alpha)\rho_L V_L i_L \rangle = \langle \Gamma_{uL} \rangle \qquad [4]$$

where u is the internal energy and i is the enthalpy. The general expressions for $\langle \Gamma_{uv} \rangle$ and $\langle \Gamma_{uL} \rangle$ are (Khater *et al.* 1980):

$$\langle \Gamma_{uv} \rangle = \frac{q''_w P_{hv}}{A_c} - \frac{q''_{vi} A'_i}{A_c} + \langle \Gamma \rangle u_{vi} - p \frac{\partial \langle \alpha \rangle}{\partial t} \qquad [5]$$

$$\langle \Gamma_{uL} \rangle = \frac{q''_w P_{hL}}{A_c} + \frac{q''_{Li} A'_i}{A_c} - \langle \Gamma \rangle u_{Li} - p \frac{\partial \langle 1-\alpha \rangle}{\partial t} \qquad [6]$$

where q'' is the heat flux, P_h is the heated perimeter, A_c is the cross sectional area, A_i' is the interface area per unit length and p is the pressure. In [5] and [6], the terms representing internal heating and dissipation at the interface have been ignored.

Lagrangian drift-flux formulation

For the reasons already given, a generalization of the drift-flux formulation of Zuber & Staub (1966) was adopted. The formulation contains the distribution parameter C_0, which accounts for void and velocity variations across the cross-section, and the vapour drift velocity V_{vj}, where

$$C_0 \equiv \langle \alpha j \rangle / \langle \alpha \rangle \langle j \rangle \quad [7]$$

$$V_{vj} \equiv V_v - j; \quad j \equiv \langle \alpha V_v \rangle + \langle (1-\alpha) V_L \rangle \quad [8]$$

where j is the superficial velocity. In addition, many other parameters (Khater *et al.* 1980) arises when [1]–[6] are converted to the drift-flux formulation. To obtain a tractable set of equations, the following approximations are introduced *within a particular interval, or subdomain, in time and space*:

(1) Internal heating, the work done by interfacial shear stresses and, as already stated, the conversion of kinetic and potential energy to internal energy are all ignored.

(2) the enthalpies of the two phases are uniform over the cross-sections.

(3) The density of the two phases, the latent heat of vaporization, C_0, and V_{vj} are all constant (over the subdomain). The values of the latter two are obtained from correlation equations, as functions of α alone.

(4) The interface between vapour and liquid remains at saturation.

(5) All averages of products (except for $\langle \alpha j \rangle$ in [7]) are approximated as the product of the averages. Thus, for example, $\langle \alpha i_v \rangle$ is approximated as $\langle \alpha \rangle \langle i_v \rangle$.

Even with approximation 5, there are three propagation velocities, V_x, in equations of the form $\frac{\partial x}{\partial t} + V_x \frac{\partial x}{\partial z} = \Gamma_x$. The liquid and vapour enthalpies are propagated at velocities V_L and V_v respectively, and void is propagated at V_α, where

$$V_\alpha = C_0 \langle j \rangle + V_{vj}. \quad [9]$$

Although not an essential approximation, the propagation velocities for i_v and i_L have, for simplicity, been both approximated by V_α. While this is not true in general, for problems in which the drift-flux formulation is generally satisfactory V_{vj} will be small so that little error is introduced. There are situations however (e.g. counter current or stratified flows) where this approximation will not be adequate. Further discussion of the validity of the drift flux formulation and the assumed equality of the propagation velocities may be found in Khater *et al.* (1980).

As pointed out in the Introduction, the effect of pressure gradients along the channel have been ignored. Pressure would normally be specified as a boundary condition at the inlet or outlet and this pressure is then taken as the system pressure. Such a treatment implies that the pressure propagates at infinite velocity.

Equations [1]–[6] can be transformed to a set of drift-flux equations (Khater *et al.* 1980) which become greatly simplified when the above approximations are introduced. Because of space limitations, only the resulting equations will be reported here in terms of the following non-dimensional parameters:

$$\begin{aligned} z^* &= \frac{z}{L}; \quad \tau = \frac{t V_a}{L}; \quad i_L^* = \frac{\langle i_L \rangle}{\lambda_0}; \quad i_G^* = \frac{\langle i_G \rangle}{\lambda_0} \\ \alpha^* &= \frac{C_0 \Delta\rho \langle \alpha \rangle}{\rho_L}; \quad \Gamma^* = \frac{C_0 \Delta\rho L}{\rho_L \rho_G V_a} \langle \Gamma \rangle; \quad V_\alpha^* = \frac{V_\alpha}{V_a} \end{aligned} \quad [10]$$

where V_a is a reference velocity, λ_0 is the reference latent heat of vaporization and L is a reference length (usually the channel length). The equations resulting from combining [1] and [2] are

$$\frac{D\alpha^*}{D\tau} = (1-\alpha^*)\Gamma^* \qquad [11]$$

$$V_\alpha^* = \frac{Dz^*}{D\tau} = (V_\alpha^*)_0 + C_0 \int_0^{z^*} \left(\frac{\Gamma^*}{C_0}\right) dz^* \qquad [12]$$

where $(V_\alpha^*)_0$ is the specified instantaneous value of V_α^* at $z^* = 0$. The following vapour generation equation is obtained by adding [3] and [4] and combining the result with [1] and [2], assuming that any vapour produced remains at saturation conditions:

$$\Gamma^* = \frac{1}{(i_G^* - i_L^*)} \cdot \frac{C_0 \Delta\rho L}{\rho_L \rho_G V_a} \cdot \left\{ \frac{q_w'' P_h}{A_c \lambda_0} + \frac{\dot{P}}{\lambda_0} - \frac{\rho_l \rho_G \dot{P} C_G}{\lambda_0 C_0 \Delta\rho} \alpha^* - \frac{\rho_L V_a}{L}\left(1 - \frac{\rho_L \alpha^*}{C_0 \Delta\rho}\right) \frac{Di_L^*}{D\tau} \right\} \qquad [13]$$

where $\dot{P} \equiv \dfrac{\partial p}{\partial t}$.

The propagation equation for liquid enthalpy, obtained from [3] and [5] is:

$$\frac{Di_L^*}{D\tau} = \frac{L}{\rho_L V_a} \cdot \frac{1}{\left(1 - \dfrac{\rho_L \alpha^*}{C_0 \Delta\rho}\right)} \cdot \left\{ \frac{q_w'' P_{hL}}{A_c \lambda_0} + \frac{q_{Li}'' A_i'}{A_c \lambda_0} - \frac{\rho_L \rho_G V_a}{C_0 \Delta\rho L} \Gamma^* (i_j^* - i_L^*) + \left(1 - \frac{\rho_L \alpha^*}{C_0 \Delta\rho}\right) \frac{\dot{P}}{\lambda} \right\} \qquad [14]$$

Under the conditions just mentioned, that the vapour produced remains at saturation conditions, the propagation equation for vapour enthalpy becomes:

$$\frac{Di_G^*}{D\tau} = C_G \dot{P} L / (V_a \lambda_0) \qquad [15]$$

Solution procedure

The fluid in the channel is initially divided into NELM elements, and a time step $\Delta\tau$ is chosen. The FAST procedure uses analytical solutions to the above equations, which are valid over the spatial subdomain occupied by the element and over the time $\Delta\tau$, to find the liquid enthalpy, the void, and the position of each element at the end of the time step. Some iteration is required because the analytical solutions used for different variables are coupled. The element, or some elements, near outflow boundaries may leave during the $\Delta\tau$ time interval. In addition, one or more elements are made to enter at the inflow boundary, or boundaries, during $\Delta\tau$. The final position and properties of each element at the end of the time step are the initial conditions for the next time step. The number of elements that are required, and the allowable time step, are dictated by the extent of the spatial and temporal domains over which the analytical solutions are valid. For the analytical solutions used, often very few elements and very large $\Delta\tau$'s were sufficient to make the global solution independent of NELM and $\Delta\tau$.

Before deriving the analytical solutions, it is necessary to describe the subscripting used. The time-vs-displacement trajectories of fluid elements in the channel are sketched in figure 1. At time level i the centres of the elements in the channel are shown ($(i, 1), (i, 2), \ldots, (i, j), \ldots (i, NELM)$). At τ_{i+1}, $\Delta\tau$ later, their positions have changed as shown; one element has been "lost" through the $z^* = 1$ boundary and two have been added through $z^* = 0$. Attention is focussed on the jth element in the time interval $\tau_i \leq \tau \leq \tau_{i+1}$. A doubly subscripted variable (e.g. α_{ij}^*) refers to its value at the point (i, j); a single subscript (e.g. α_j^*) refers to the value of the variable at a general time τ within the time interval under consideration.

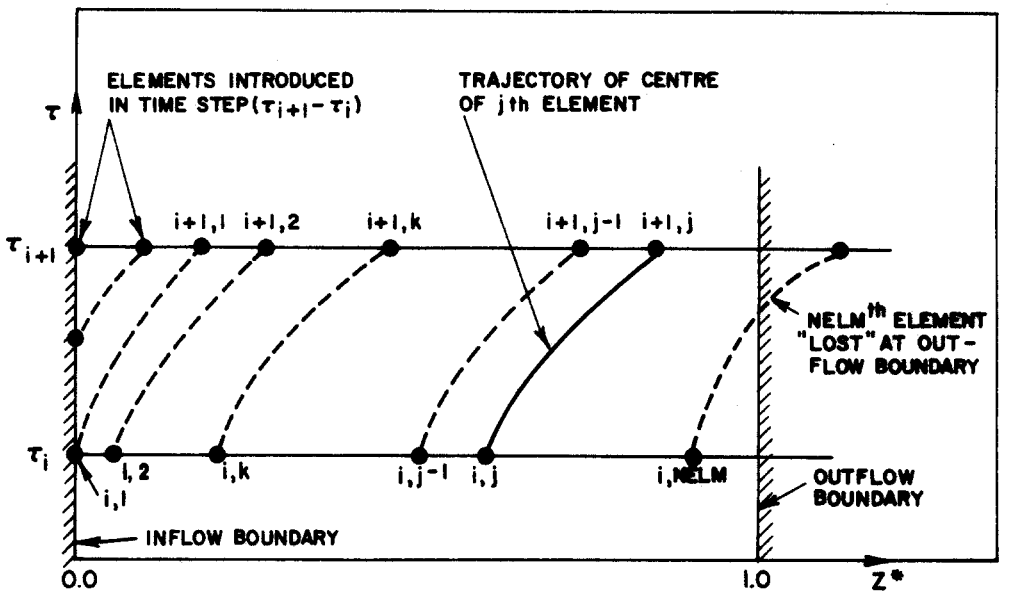

Figure 1. Element positions at the beginning and end of time step.

Some additional simplifications are introduced into [12]–[14], which are valid over restricted subdomains of τ and z^*, in order to obtain the required analytical solutions. For a particular element over the $\Delta\tau$ interval, these are:

(6) The inlet velocity is a linear function of time over the $\Delta\tau$ interval, i.e. $(V_\alpha^*)_0 = (V_\alpha^*)_{0i} + (\dot{V}_\alpha^*)_0(\tau - \tau_i)$, where $(V_\alpha^*)_{0i}$ is the inlet velocity at τ_i and $(\dot{V}_\alpha^*)_0$ is the average rate of change of $(V_\alpha^*)_0$ with τ over the $\Delta\tau$ interval.

(7) The saturation enthalpies of both phases are linear functions of pressure (over the subdomain).

(8) The wall heat flux is constant (over the subdomain).

(9) In calculating Γ^* from [13], $i_G^* - i_f^*$ is replaced by the average value for the jth element over the time interval (i.e. $(i_G^* - i_f^*)$) and $Di_f^*/D\tau$ is replaced by $((i_f^*)_{i+1j} - (i_f^*)_{ij})/\Delta\tau$.

With these approximations, [11] and [13] can be combined to give, for the jth element,

$$\frac{D\alpha_j^*}{D\tau} = a_\alpha + b_\alpha \alpha_j^* + c_\alpha (\alpha_j^*)^2 \qquad [16]$$

where a_α, b_α and c_α are constants over a particular subdomain (see appendix A.). The initial conditions are $\alpha_j^* = \alpha_{ij}^*$ at $\tau = \tau_i$. The solution to [16], which depends on the sign of $\Delta_\alpha = 4a_\alpha c_\alpha - b_\alpha^2$, is (Gradshteyn & Ryzhik 1965):

$$\alpha_j^* = \begin{cases} \dfrac{b_\alpha - \sqrt{-\Delta_\alpha} - (b_\alpha + \sqrt{-\Delta_\alpha})\exp[\sqrt{-\Delta_\alpha}(\tau + c_1)]}{2c_\alpha(\exp[\sqrt{-\Delta_\alpha}(\tau + c_1)] - 1}; & \Delta_\alpha < 0 \\ -\dfrac{1}{2c_\alpha}\left(b_\alpha + \dfrac{2}{\tau + c_2}\right); & \Delta_\alpha = 0 \\ \dfrac{1}{2c_\alpha}\left\{\sqrt{\Delta_\alpha}\tan\left[\dfrac{\sqrt{\Delta_\alpha}(\tau + c_3)}{2} - b_\alpha\right]\right\}; & \Delta_\alpha > 0. \end{cases} \qquad [17]$$

The details appear in appendix A.

The position of this element at $\tau > \tau_i$ is obtained from [12]. The integral is approximated as

$$\int_0^{z_j^*} \left(\frac{\Gamma^*}{C_0}\right) dz^* = \sum_{k=1}^{j-1} \left(\frac{\bar{\Gamma}^*}{C_0}\right)_k (z_{ik+1}^* - z_{ik}^*) + \left(\frac{\bar{\Gamma}^*}{C_0}\right)_j (z_j^* - z_{ij}^*)$$

where $\left(\frac{\bar{\Gamma}^*}{C_0}\right)_k$ is the value of $\left(\frac{\Gamma^*}{C_0}\right)$ at $\tau_i + \Delta\tau/2$ averaged over the range $z_{ik}^* \le z^* \le z_{ik+1}^*$. Introducing approximation 6 and carrying out the integration yields the following position of the jth element as a function of τ:

$$z_j^* = z_{ij}^* + \frac{\exp[\bar{\Gamma}_j^*(\tau - \tau_i)] - 1}{\bar{\Gamma}_j^*} \left\{(V_\alpha^*)_0 + \frac{(\dot{V}_\alpha^*)_0}{\bar{\Gamma}_j^*} + C_{0_j} \sum_{k=1}^{j-1} \left(\frac{\bar{\Gamma}^*}{C_0}\right)_k (z_{ik+1}^* - z_{ik}^*)\right\}$$
$$- \frac{(\dot{V}_\alpha^*)_0}{\bar{\Gamma}_j^*} \{\tau - \tau_i \exp(\bar{\Gamma}_j^*(\tau - \tau_i))\}. \qquad [18]$$

When $\bar{\Gamma}_j^*$ approaches zero, the exponentials in [18] are replaced by their Taylor series expansion. Some details related to the derivation of [18] are contained in appendix A.

The solution to the liquid enthalpy equation [14] for the jth fluid element is now described. Substituting [13] for Γ^* into [14], defining $(\overline{C_p})_L = (i_f - \langle i_L \rangle)/(T_s - T_L)$, replacing α^* over the jth element during the interval $\Delta\tau$ by $\overline{\alpha_j^*}$, and using approximation 7, the liquid enthalpy equation becomes

$$\frac{Di_{Lj}^*}{D\tau} = k_1 + k_2\tau + k_3 i_{L_j}^* + k_4\tau^2 + k_5\tau i_{L_j}^* + k_6(i_{L_j}^*)^2. \qquad [19]$$

The values of k_1 to k_6 are constants over the jth element and over $\Delta\tau$. The series solution to this equation is

$$i_{L_j}^* = \sum_{n=1}^{\infty} nA_n\tau^{n-1} \Big/ \left\{-k_6\left(1 + \sum_{n=1}^{\infty} A_n\tau^n\right)\right\} \qquad [20]$$

Again, the details related to [19] and [20] are contained in appendix A.

For the element closest to $z^* = 0$, and starting at τ_i, the closed-form solutions above are used to determine the position and properties of the element at τ_{i+1}. This process is then repeated for each element in the channel. Presuming that solutions have already been obtained for the first $j - 1$ elements, the solution for the jth element is established as follows:

Step 1. The enthalpy of the liquid in the jth element is found from [20]. The constants contain the average void $\overline{\alpha_j^*}$ which is not yet known. In the present study it was found that α_j^* could be replaced, without appreciable error, by α_{ij}^*. This avoids the need for iteration.

Step 2. With $Di_j^*/D\tau$ known (see approximation 9), the constants in [17] can be evaluated and α_{i+1j}^* found.

Step 3. The values of $(i_j^*)_{i+1j}$, Γ_{i+1j}^* and α_{i+1j}^* (i.e., at τ_{i+1}) are now known and the position of the element at τ_{i+1} is sought. This is found from [18]. Iteration is needed because the averaged quantities denoted by $(\bar{\ })$ depend on the final position of the element (i.e. on z_{i+1j}^*).

These solutions then serve as initial conditions for the advancement of the solution over the next time step.

APPLICATIONS

The predictions obtained by applying the FAST procedure to several problems are now reported. In some cases these predictions are compared with experimental data. In other cases, contrived problems, for which there are no experimental data available, have been used to

demonstrate the capabilities of FAST. In the first category, the accuracy of the predictions will depend on the expressions chosen for C_0, V_{vj}, A_i', etc., so that care is needed in their selection. When only a demonstration of capability is desired, the particular choice becomes of much less importance. The physical phenomena in the problems chosen involve subcooled boiling, flow reversal and blowdown.

Steady subcooled boiling

For subcooled boiling predictions, the following expression for C_0, developed by Hancox & Nicoll (1971), was used:

$$C_0 = 1 + C_{01}(1 - \langle\alpha\rangle)$$
$$C_{01} = 1.164 - 3.655P_{red} + 3.670P_{red}^2. \quad [21]$$

For the drift velocity, V_{vj}, the following simple approximation was used

$$V_{vj} \approx 0. \quad [22]$$

When the liquid is highly subcooled, the vapour is confined to a region very near the wall. Applying the approximation due to Hancox & Nicoll (1971), the vapour is modelled as forming an annular ring adjacent to the wall so that the liquid-vapour interface area is

$$A_i' = \pi D_h \sqrt{\langle 1 - \alpha \rangle}. \quad [23]$$

Direct heating of the liquid is ignored ($P_{hl} = 0$ in [14]). The heat transfer from the vapour interface to the subcooled liquid is given by

$$q_{Li}'' = h_{Li}(i_f - \langle i_L \rangle)/(\overline{C_p})_L. \quad [24]$$

The value of h_{li} was taken as the maximum of the two quantities inside the brackets in the equation

$$h_{Li} = \left[0.40 \frac{k_f}{D_h} \mathrm{Re}^{0.622} \mathrm{Pr}^{0.4}, 455 \cdot \frac{k_f}{D_h}\right]_{MAX}. \quad [25]$$

The first expression, intended for high velocities, was proposed by Hancox & Nicoll (1971), while the second, for low velocities, was taken from the work of Saha & Zuber (1974).

Upstream of the "point of significant vapour generation", $\langle\alpha\rangle$ is assumed to be zero, and the heat transfer that can be accepted by the liquid, without requiring superheat at the wall, is given by [24]. if $q_w'' < q_{Li}''$ all the wall heat transfer is passed to liquid, thereby reducing its subcooling, $A_i' = 0$, and $P_{hL} = \pi D_h$ in [14]. The "point of significant vapour generation" is located where $q_w'' = q_{Li}''$. Farther downstream, unless q_w'' is sharply reduced, subcooled boiling occurs.

The assumptions embodied in [23] and [25] are no longer valid if $\langle\alpha\rangle$ is large. However, when $\langle\alpha\rangle$ has become large, the subcooling of the liquid must be small (i.e. $q_{Li}'' A_i' \approx 0$) so that large percentage errors in $q_{Li}'' A_i'$ become inconsequential.

Comparisons between predicted and measured void distributions have been made for a total of 63 experiments reported by Egen *et al.* (1957), Maurer (1956), Christensen (1961), Marchaterre *et al.* (1960) and Foglia *et al.* (1961). The experiments covered a wide range of flow conditions and inlet subcooling, but were steady in all cases. (There are, unfortunately, no extensive and reliable measurements of axial void distributions for unsteady flows.) A representative sample of the comparisons are presented in figure 2. The root mean square

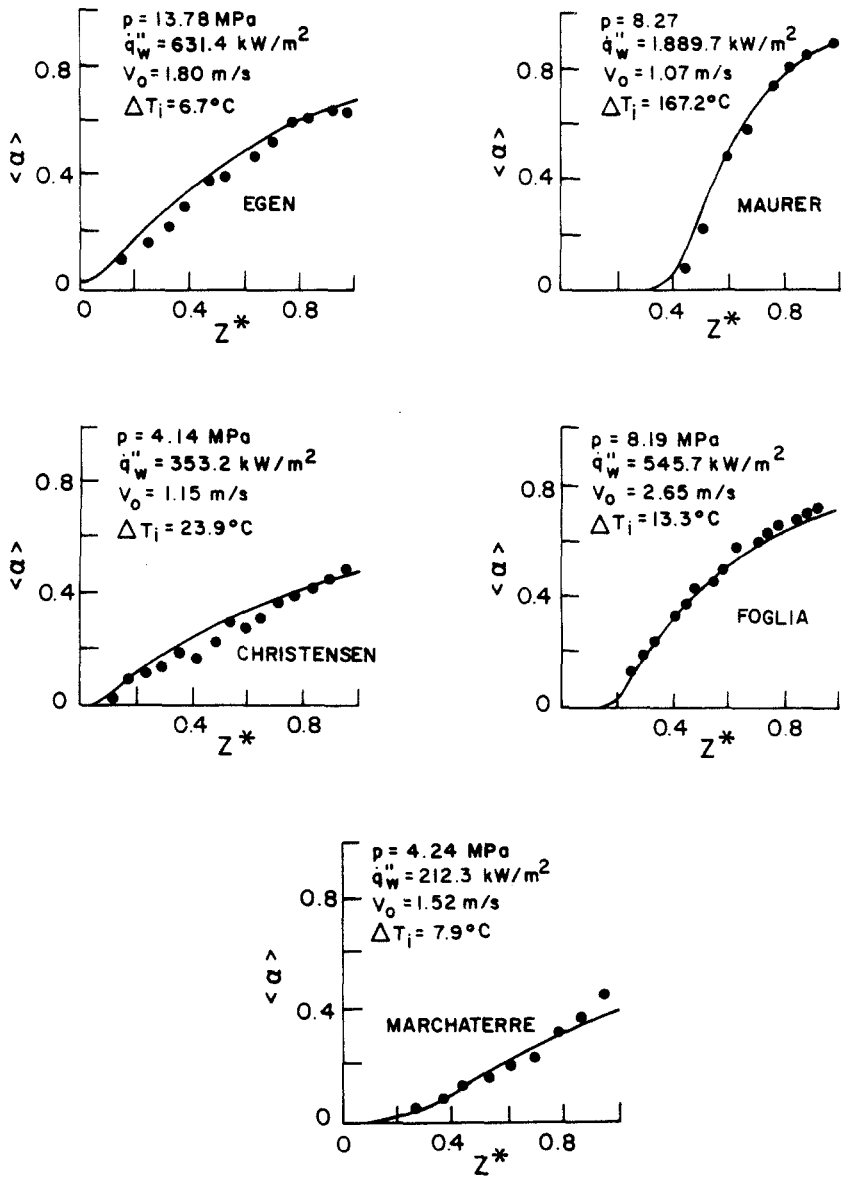

Figure 2. A representative sample of the comparisons made between predicted and measured α under subcooled boiling conditions.

deviation between measured and predicted void, including all 63 cases, was about 5%. The model, therefore, adequately represents the departures from thermal equilibrium which occur in this type of flow.

The results shown in figure 2 were obtained using 10 elements in the channel. The CPU time required for each run was about 1 second on an IBM 360–75 computer. Predictions for some of the cases were repeated using respectively, 6, 32 and 51 elements in the channel. The results using 10, 32 and 51 were virtually identical, while the predictions with 6 elements, including the point of incipient boiling, were only slightly different.

Tests were also done with h_{Li} values from 0.25 to 4.0 times the values given in [25], but the predictions based on this equation agreed most closely with measurement. Predictions were repeated for C_0 values of 1.0, 1.13, 1.2, and 1.5, but [21] yielded superior agreement with experiment. Finally, other drift velocities were used, but again, the original proposal [22] was found to be the most suitable.

Flow reversal

The direction of flows in a channel may be reversed, either locally or everywhere, as a consequence to two distinct mechanisms: (1) reversal of the channel-end velocity; (2) a change in the mean density of the fluid in the channel, either through vaporization or condensation (density driven reversal). Of course, in any given problem, both mechanisms may be operative. Two contrived problems are presented below to cover the above-mentioned situations. Equations [21]–[25] are used to determine C_0, V_{vj}, and the inter-phase heat transfer.

The first problem considered is a simple type of boundary-condition driven flow reversal. The physical system consists of a constant area channel with constant and uniform heat flux. At the start of the transient, the inlet velocity is decreased at a uniform rate. A slight depressurization is assumed to illustrate that simultaneous boiling and flashing poses no problem. The initial and boundary conditions, together with the predictions of axial void distributions and local mixture velocity, are shown in figure 3. The predictions are plausible for the postulated conditions and, most importantly, display the capability of the FAST procedure to deal with inlet flow reversal and with situations in which the flow may be outward from the channel at both ends.

In the previous example, vapour was generated both through subcooled boiling and through flashing. The second problem is chosen to illustrate flow reversal due to condensation. Again, a constant area channel is considered, but at the start of the transient both the inlet flow and the wall heat flux are abruptly set to zero. Since in the upstream portion of the two-phase region the

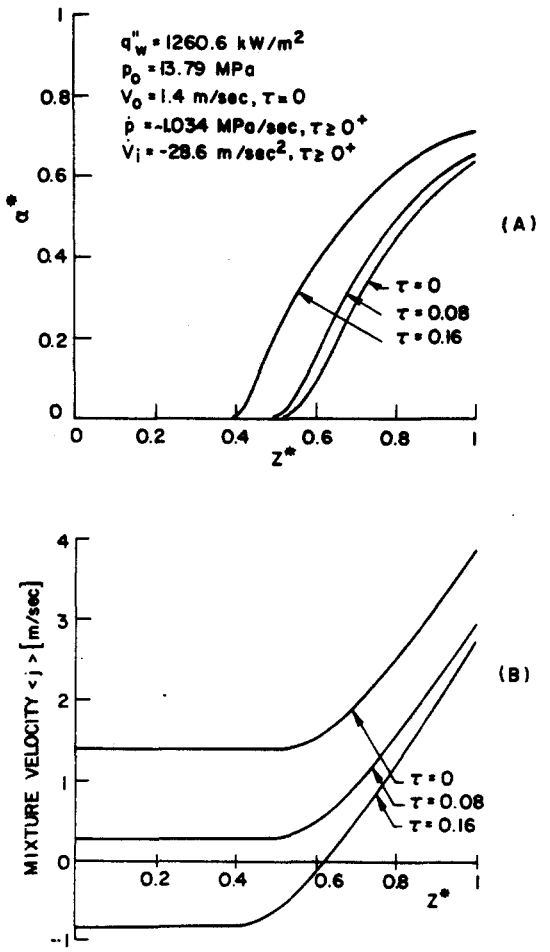

Figure 3. Predicted axial distributions of void (a) and mixture velocity (b) for a linear decrease in inlet velocity and system pressure for a contrived problem.

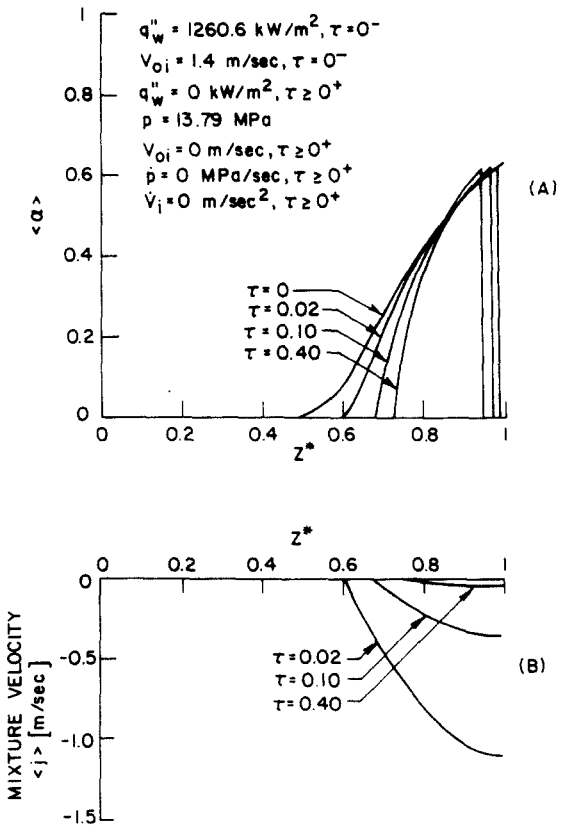

Figure 4. Predicted axial distributions of void (a) and mixture velocity (b) for a sudden flow stoppage at the inlet and power shutdown.

liquid phase is still significantly subcooled, heat will be transferred from the interface to the liquid, with resultant condensation of the vapour. Farther downstream, where the liquid has nearly reached saturation, no inter-phase heat transfer and no condensation will occur. For these fluid elements, the void and mixture density should remain nearly constant. These expectations are consistent with the predicted axial void distributions shown in figure 4. With regard to the former, we note that condensation is indeed predicted in the upstream portions of the initial two-phase region and that little change in void is predicted for those fluid elements which initially had a relatively high void. Because of the upstream condensation, however, these fluid elements are drawn back toward the region of condensation and it becomes necessary to specify the state of the fluid which is drawn into what was originally the downstream end of the channel. In the present problem, this has been chosen as saturated liquid. Two observations are appropriate here. First, it can be seen that the discontinuity between the void of the fluid originally in the channel and that of the new fluid drawn into the channel is not diffused; it is a characteristic of the present method that the "artificial" or "false" diffusion (Raithby 1975) associated with Eulerian finite-difference methods is not present. Second, it has been said that the state of the fluid drawn into the system must be specified. This is true, of course, of any prediction procedure, only in the present case there is no need to devise different procedures for inflow and outflow boundaries. Since the prediction procedure is Lagrangian, the initial states of the particular fluid elements being tracked are required for the integration over each time step.

The features discussed above with regard to axial void distributions are also displayed by the mixture velocity distributions in figure 4. This velocity is zero at the beginning of the two-phase region and becomes, due to condensation, increasingly negative with increasing

distance from the inlet. As the end of the channel is approached, however, condensation and hence mean density change diminish; this is reflected in the zero slope of the velocity distribution at values of z^* near unity.

The results reported in figure 3 and 4 were obtained using 30 elements in the channel, and with $\Delta\tau = 0.02$. About 20 seconds of CPU time were required to obtain the results shown at $\tau = 0.40$.

Space limitations preclude the presentation of the results of other contrived problems (Khater *et al.* 1980). In some cases, conditions were such that α^* varied sharply along the channel. For one of these cases extensive numerical experiments were performed to determine the dependence of solution accuracy on $\Delta\tau$ and the number of elements used. It was found that very large time steps could be taken before accuracy became seriously degraded. In addition, the minimum number of elements was established more by the need to have enough points to draw a continuous curve than by a degradation in accuracy of the computed points. The FAST method gave roughly equivalent accuracy to that obtained by a simple explicit finite difference method (of solving [11], [12] and [14]) with an order of magnitude less computational effort. (For an RMS error in the predictions of 4%, FAST required 15 CPU seconds. The explicit solution method required 138 CPU seconds for an RMS error of 3.2%.)

Blowdown

Mass holdup measurements have been reported by Primoli & Hancox (1976) for blowdown from an apparatus consisting of an unheated feeder section, a heated section of larger diameter, and an unheated riser of still larger diameter. At the high initial pressure the feeder flow was highly subcooled, while fluid in the riser was near saturation. At the start of the transient the riser outlet was blocked, the feeder was opened to atmosphere, and the wall heat flux was maintained constant in the middle section, resulting in flow reversal and discharge back through the feeder. Because of the higher fluid temperature in the riser, flashing may be expected to occur first in the riser as well as at the open end of the feeder. As the system pressure becomes lower, the region of flashing may be expected to move into the heated section. The data reported included the mass holdup in the three sections and pressures at various locations in the apparatus. Mass holdup measurements are reporduced in figure 5.

The boundary conditions applied in the model were zero velocity at the riser exit, and a system pressure equal to the average pressure in the riser section. The heat transfer model was the same as that used in the subcooled boiling application. Because of the rapid depressurization, the flow will tend to be homogeneous and thus a value of unity has been assigned to C_0.

Turning now to the comparison of prediction and measurement in figure 5, it is seen that in general the agreement is good. The only significant discrepancy occurs in the feeder mass holdup at approximately 0.4 seconds where the experimental values are lower than the predicted values. This is likely a consequence of the assumption of spatially uniform pressure which underestimates flashing in the feeder. With this exception, the agreement is good. Mass balances on the apparatus showed that FAST conserved mass closely (within 1%) throughout the transient.

In these predictions a large number of elements (125) were initially required in the channel because so many of them were carried out through the feeder with the fluid. The predictions required 8 CPU seconds.

CLOSURE

This paper has described the FAST method of solving the drift-flux equations for transient two-phase flows, including the effect of unequal phase temperatures (subcooled liquid-saturated vapour). Solutions have been reported to problems for which experimental data are available and to certain contrived problems that were designed to exercise fully the capabilities of FAST.

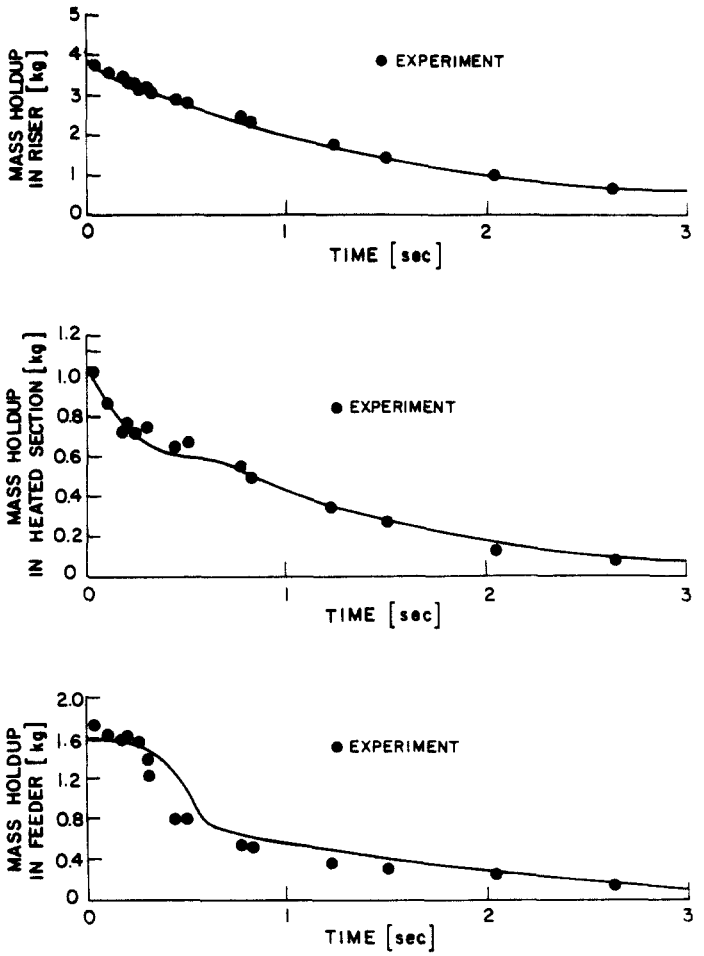

Figure 5. A comparison of predicted and measured mass holdup in the riser (a), heated section (b) and feeder (c) for Standard Problem 3 (Promoli & Hancox 1976).

The physical phenomena included subcooled boiling, flow reversal and blowdown. The main advantages of the FAST method are the small computational effort that is required to obtain accurate solutions, and the numerical stability of the scheme.

The limitations of the present model were described in the Introduction. Of these, the need to specify the drift velocity is particularly restrictive. By including in the equation set an additional equation, derived from the liquid and vapour momentum equations by eliminating the pressure gradient, it is possible to calculate the difference between the phase velocities. An analytical solution to this equation has been found, in the spirit of the FAST method, and solutions to unequal velocity problems have been obtained (Khater & Raithby 1981). Although this was only a preliminary study, it showed that some of the advantages of FAST are lost (a) because more iterations are required for each time step, and (b) because at least one of the phase velocities is different from the velocity in the convective derivative that establishes how the control volumes move through the system. The first results in increased computational effort, while the second results in a smearing of steep profiles, similar to the effects of false diffusion. Despite these shortcomings, this method would likely prove competitive with other two-fluid solution methods.

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NOMENCLATURE

A_c cross-sectional area, m^2
A_i' interface area per unit length, m
C_f di_f/dp, J/kg MPa
C_G di_G/dp, J/kg MPa
C_0 $\langle \alpha j \rangle / \langle \alpha \rangle \langle j \rangle$; Zuber's distribution parameter, dimensionless
C_p specific heat, J/kg K
D_h hydraulic diameter, m
h coefficient of heat transfer, W/m^2 K
i enthalpy, J/kg
i^* $\frac{i}{\lambda_0}$, (dimensionless)
j superficial velocity, m/s
k thermal conductivity, W/m K
L length, m
P perimeter, m
Pr Prandtl number
p pressure, MPa
$\dot{p}$ $\partial p/\partial t$, MPa/sec
q'' heat flux, W/m^2
Re $\rho_f j D_h/\mu_f$, Reynolds number, dimensionless
T temperature, K
ΔT_i inlet subcooling, K
t time, sec
u internal energy, J/kg
V velocity, m/sec
V_{vj} $V_v - j$, (m/sec)
V_α $c_0\langle j \rangle + V_{vj}$, m/sec
V_α^* V_α/V_a, dimensionless
$\dot{V}_\alpha^*$ average rate of change of $(V_\alpha^*)_0$ with τ
z axial position, m
z^* z/L, dimensionless
α void fraction, dimensionless
α^* $c_0\Delta\rho\langle\alpha\rangle/\rho_L$, dimensionless
Γ rate of vapour generation, kg/m^3 sec
Γ^* $(c_0\Delta\rho L/\rho_L\rho_G V_a)\Gamma$, dimensionless
λ latent heat of vaporization, J/kg
λ_0 reference latent heat of vaporization, J/kg
ρ density, kg/m^3
$\Delta\rho$ $\rho_L - \rho_G$, kg/m^3
τ tV_a/L, dimensionless
$\Delta\tau$ $\tau - \tau_i$, dimensionless

Subscripts

a arbitrary value
f saturated liquid

G saturated vapour
i initial
ij initial for the jth element
j the jth element
L liquid
L_i liquid interface
vi vapour interface
0 channel inlet
$0i$ initial value at the channel inlet
red reduced
s saturation
v vapour
w wall

Superscripts
n new time level
0 old time level

APPENDIX A

A.1 *Solution of the void propagation equation*

The void propagation, [16], is

$$\frac{D\alpha_j^*}{D\tau} = a_\alpha + b_\alpha \alpha_j^* + c_\alpha (\alpha_j^*)^2 \qquad [A1]$$

where

$$a_\alpha = \frac{c_0 \Delta\rho L}{\rho_L \rho_G V_a} \frac{\frac{q_w'' P_h}{A_c} + \frac{\dot{P}}{\lambda_0} - \frac{\rho_L V_a}{L}\left(\frac{\Delta i_L^*}{\Delta\tau}\right)}{(\overline{i_G^* - i_L^*})}$$

$$c_\alpha = -\frac{1}{\rho_G} \frac{\left[\rho_L\left(\frac{\Delta i_L^*}{\Delta\tau}\right) - \rho_G \frac{C_G \dot{P} L}{V_a \lambda_0}\right]}{(\overline{i_G^* - i_L^*})} \qquad [A2]$$

$$b_\alpha = -(a_\alpha + c_\alpha).$$

Subject to the boundary condition $\alpha^* = \alpha_{ij}^*$ at $\tau = \tau_i$, [17] is the solution to [A1] where

$$\Delta_\alpha = 4a_\alpha c_\alpha - b_\alpha^2$$

$$c_1 = \frac{1}{\sqrt{-\Delta_\alpha}} \ln \frac{b_\alpha + 2c_\alpha \alpha_{ij}^* - \sqrt{-\Delta_\alpha}}{b_\alpha + 2c_\alpha \alpha_{ij}^* + \sqrt{-\Delta_\alpha}} - \tau_i$$

$$c_2 = -\left(\tau_i + \frac{2}{b_\alpha + 2c_\alpha \alpha_{ij}^*}\right) \qquad [A3]$$

$$c_3 = \frac{2}{\sqrt{\Delta_\alpha}} \tan^{-1}\left(\frac{b_\alpha + 2c_\alpha \alpha_{ij}^*}{\sqrt{\Delta_\alpha}}\right) - \tau_i$$

A.2 *Solution of the space-time differential equation*

Equation [12] establishes the position of each element at the end of the time step. Rewriting this equation for the jth element

$$\frac{dz_j^*}{d\tau} = V_{a_j}^* = (V_\alpha^*)_0 + C_{0_j} \int_0^{z_j^*} \left(\frac{\Gamma^*}{C_0}\right) dz^* \qquad [A4]$$

As discussed in the text, the integral can be approximated by

$$\int_0^{z_j^*} \left(\frac{\Gamma^*}{C_0}\right) dz^* = \sum_{k=1}^{j-1} \left(\frac{\tilde{\Gamma}^*}{C_0}\right)_k (z_{ik+1}^* - z_{ik}^*) + \left(\frac{\tilde{\Gamma}^*}{C_0}\right)_j (z_j^* - z_{ij}^*). \qquad [A5]$$

Equation [A4] thus becomes, using approximation 6 in the text,

$$\frac{dz_j^*}{d\tau} - \tilde{\Gamma}_j^* z_j^* = (V_\alpha^*)_{0i} + (\dot{V}_\alpha^*)_0(\tau - \tau_i) + c_j \qquad [A6]$$

where

$$c_j = C_{0_j} \sum_{k=1}^{j-1} \left(\frac{\tilde{\Gamma}^*}{C_0}\right)_k (z_{ik+1}^* - z_{ik}^*) - (\tilde{\Gamma}^*) z_{ij}^*.$$

Equation [A6] is a first order, linear, non-homogeneous ordindary differential equation; its solution given by

$$z_j^* \, e^{\int -(\tilde{\Gamma}_j^*) d\tau} = \int e^{\int -(\tilde{\Gamma}_j^*) d\tau} ((V_\alpha^*)_{0i} + (\dot{V}_\alpha^*)_0(\tau - \tau_i) + c_j) \, d\tau + c_4$$

Subject to the boundary condition $z^* = z_{ij}^*$ at $\tau = \tau_i$, the above integration yields [18] in the text.

A.3 *Solution of the liquid enthalpy propagation equation*

Equation [19] is the liquid enthalpy propagation equation

$$\frac{Di_{L_j}^*}{D\tau} = k_1 + k_2\tau + k_3 i_{L_j}^* + k_4\tau^2 + k_5\tau i_{L_j}^*) + k_6(i_{L_j}^*)^2 \qquad [A7]$$

Defining the constants

$$A_L = \left(\frac{q_w'' P_h}{A_c} + \dot{P}\left(1 - \frac{\rho_G \rho_L}{\Delta\rho C_0} C_G \tilde{\alpha}^*\right)\right) \Big/ \lambda$$

$$B_L = \left(\frac{q_w'' P_{hL}}{A_c} + \left(1 - \frac{\rho_L}{c_0 \Delta\rho} \tilde{\alpha}^*\right)\dot{P}\right) \Big/ \lambda$$

$$C_L = \frac{h_{Li} A_j' \lambda_0}{A_c \bar{C}_{pL} \lambda} \qquad [A8]$$

$$D_L = -C_f \dot{P} \frac{L}{V_\alpha \lambda_0}$$

$$E_L = -C_G \dot{P} \frac{L}{V_\alpha \lambda_0}; \quad F_L = \frac{L}{V_\alpha \rho_L \left(1 - \frac{\rho_L}{C_0 \Delta\rho} \tilde{\alpha}^*\right)}$$

the constants k_1 through k_6 are

$$k_1 = (-A_L i^*_{\gamma_i} + B_L i^*_{\delta_i} + C_L i^*_{\delta_i} i^*_{\gamma_i}) F_L$$

$$k_2 = (D_L A_L - E_L B_L - C_L D_L i^*_{\delta_i} - C_L E_L i^*_{\gamma_i}) F_L$$

$$k_3 = (A_L - B_L - C_L (i^*_{\gamma_i} + i^*_{\delta_i})) F_L$$

$$k_4 = C_L D_L E_L F_L$$

$$k_5 = (D_L + E_L) C_L F_L$$

$$k_6 = C_L F_L. \qquad \text{[A9]}$$

Equation [A7] has the form

$$(i^*_L)' + P(\tau) i^*_L + Q(\tau)(i^*_L)^2 = R(\tau)$$

which is a Ricatti equation (Gradshteyn and Ryzhik, 1975). Introducing the transformation $i^*_L = \dfrac{u'}{uQ}$, [A7] becomes

$$u'' + (b_0 + b_1 \tau) u' + (b_2 + b_3 \tau + b_4 (\tau)^2) u = 0 \qquad \text{[A10]}$$

where

$$b_0 = -k_3; \quad b_1 = -k_5; \quad b_2 = k_1 k_6; \quad b_3 = k_2 k_6; \quad b_4 = k_4 k_6. \qquad \text{[A11]}$$

To solve [A10], assume a series solution of the form

$$u = \sum_{n=0}^{\infty} a_n (\tau)^{m+n}. \qquad \text{[A12]}$$

Differentiating twice, and substituting into [A10] yields

$$\sum_{n=0}^{\infty} a_n (n+m)(n+m-1)(\tau)^{n+m-2} + b_0 \sum_{n=1}^{\infty} a_{n-1} (n+m-1)(\tau)^{n+m-2}$$
$$+ \sum_{n=2}^{\infty} [b_2 + (n+c-2) b_1] a_{n-2} (\tau)^{n+m-2} + b_3 \sum_{n=3}^{\infty} a_{n-3} (\tau)^{n+m-2}$$
$$+ \sum_{n=4}^{\infty} a_{n-4} (\tau)^{n+m-2} = 0. \qquad \text{[A13]}$$

Equating the sum of the coefficients of the different powers of τ to zero starting with the lowest power ($n = 0$) we find, from the $n = 0$ and $n = 1$ equations, that $m = 0$ and that both a_0 and a_1 are arbitrary constants. The remainder of the coefficients in the series are found in the same manner and are listed below. To find i^*_L substitute [A12] into the transformation used. This yields

$$i^*_L = \frac{\sum_{n=1}^{\infty} n a_n \tau^{n-1}}{-k_6 \sum_{n=0}^{\infty} a_n \tau^n}. \qquad \text{[A14]}$$

Dividing by a_0, and defining the coefficients $A_n \equiv \frac{a_n}{a_0}$ yields [20] in the text where A_1 is an arbitrary constant and can be found from the boundary condition $i^{\dagger}_{L_j} = i^{\dagger}_{L_{ij}}$ at $\tau = \tau_0$. In the case where $\tau_0 = 0$, $A_1 = -k_6 i^{\dagger}_{L_{ij}}$. The other coefficients are

$$A_2 = -(b_0 A_1 + b_2)/2$$

$$A_3 = -(2b_0 A_2 + (b_2 + b_1)A_1 + b_3)/6 \qquad \text{[A15]}$$

$$A_n = -((n-1)b_0 A_{n-1} + (b_2 + (n-2)b_1)A_{n-2} + b_3 A_{n-3} + b_4 A_{n-4})/(n(n-1)); \quad n \geq 4.$$